NCTM: Principles & Standards for School Mathematics (*PSSM*) Est: 2000

Equity: The NCTM standards for equity, as outlined in the **PSSM**, encourage equal access to mathematics for all students, "especially students who are poor, not native speakers of English, disabled, female, or members of minority groups."[3] The PSSM makes explicit the goal that all students should learn higher level mathematics, particularly underserved groups such as minorities and women. This principle encourages provision of extra help to students who are struggling and advocates high expectations and excellent teaching for all students.[3]

Curriculum: In the **PSSM's** curriculum section, the NCTM promotes a "coherent" curriculum, in which an orderly and logical progression increases students' understanding of mathematics and avoids wasting time with unnecessary repetition.[4] They acknowledge that the relative importance of some specific topics changes over time.[4] For example, a basic understanding of iteration is important to students who are learning computer programming, and is almost absent from 19th century textbooks. Similarly, older American math textbooks included lessons that are no longer considered important, such as rules for calculating the number of bushels of hay that could be stored in a bin of stated dimensions, because this skill was useful to farmers at that time.[5] The NCTM proposes that mathematics taught in modern classrooms be the skills that are most important to the students' lives and careers.

Teaching: In the **PSSM**, the NCTM promotes sound teaching methods, without prescribing a one-size-fits-all approach.[6] The NCTM wants teachers to be able to use their professional judgment in choosing teaching techniques. They favor professional development opportunities in both mathematics (content) and in effective teaching techniques (methods).

Learning: According to the **PSSM**, a combination of "factual knowledge, procedural facility, and conceptual understanding" is necessary for students to use mathematics.[7] While they state that 'Learning the "basics" is important,'[7] the NCTM does not consider the most simplistic forms of memorization by repetition to be *sufficient* achievement in mathematics. A good student not only understands how and when to use facts, procedures, and concepts, but he or she also wants to figure things out and perseveres in the face of challenge. The NCTM particularly deprecates attitudes in schools that suggest only certain students are capable of mastering math.

Assessment: The Assessment Standards for School Mathematics has been produced by the National Council of Teachers of Mathematics (NCTM) because we believe new assessment strategies and practices need to be developed that will enable teachers and others to assess students' performance in a manner that reflects the NCTM's reform vision for school mathematics. Our vision includes the mathematics we expect students to know and be able to use, the way they have learned it, and how their progress is to be assessed. For school assessment practices to inform educators as they progress toward this vision, it is essential that we move away from the "rank order of achievement" approach in assessment toward an approach that is philosophically consistent with NCTM's vision of school mathematics and classroom instruction.

Reference: http://www.nctm.org/

Reference: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Reference: http://www.corestandards.org/

Reference: http://www.ala.org/aasl/standards-guidelines/crosswalk

Reference: http://www.greatschools.org/worksheets/

Reference sites are a view into the History of NCTM Principles, Standards, etc… On and On…
Overview of NCTM Standards: http://en.wikipedia.org/wiki/NCTM_standards (Wiki Free Location)

The USA National Standards for SS, LA, Math, Science, Technology are included below: (All Discipline Sites)
US Mint: http://www.usmint.gov/kids/teachers/lessonplans/nationalStandards.cfm

***Content* Standards:** Instructional programs for **Pre-K to 12** should enable students to:

Number and Operations Standard:

• Understand numbers, ways of representing numbers, relationships among numbers, and number systems;
• Understand meanings of operations and how they relate to one another; and
• Compute fluently and make reasonable estimates.

Measurement Standard:

• Understand measurable attributes of objects and the units, systems, and processes of measurement; and
• Apply appropriate techniques, tools, and formulas to determine measurements.

Geometry Standard:

• Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
• Specify locations and describe spatial relationships using coordinate geometry and other representational systems;
• Apply transformations and use symmetry to analyze mathematical situations; and use visualization, spatial reasoning, and geometric modeling to solve problems.

Algebra Standard:

• Understand patterns, relations, and functions;
* from *Principles and Standards for School Mathematics*
• Represent and analyze mathematical situations and structures using algebraic symbols;
• Use mathematical models to represent and understand quantitative relationships; and
• Analyze change in various contexts.

Data Analysis and Probability:

• Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
• Select and use appropriate statistical methods to analyze data;
• Develop and evaluate inferences and predictions that are based on data; and
• Understand and apply basic concepts of probability.

Content ***(Process) Standards:*** Instructional programs for **Pre-K to 12** should enable students to:

Problem Solving Standard:

• Build new mathematical knowledge through problem solving;

solve problems that arise in mathematics and in other contexts;

• Apply and adapt a variety of appropriate strategies to solve problems; and

• Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof Standard:

• Recognize reasoning and proof as fundamental aspects of mathematics;

make and investigate mathematical conjectures;

• Develop and evaluate mathematical arguments and proofs; and

• Select and use various types of reasoning and methods of proof.

Communication Standard:

• Organize and consolidate their mathematical thinking through communication;

• Communicate their mathematical thinking coherently

and clearly to peers, teachers, and others;

• Analyze and evaluate the mathematical thinking and strategies of others; and

• Use the language of mathematics to express mathematical ideas precisely.

Connections Standard:

• Recognize and use connections among mathematical ideas;

• Understand how mathematical ideas interconnect and build on one another

to produce a coherent whole; and

• Recognize and apply mathematics in contexts outside of mathematics.

Representation Standard:

• Create and use representations to organize, record, and communicate mathematical ideas;

• Select, apply and translate among mathematical representations to solve problems; and

• Use representations to model and interpret physical, social, and mathematical phenomena.

NCTM: What are Curriculum *Focal Points* for Mathematics Established: 2006

Curriculum focal points are important mathematical topics for each grade level, pre-K–8. These areas of instructional emphasis can serve as organizing structures for curriculum design and instruction at and across grade levels. The topics are central to mathematics: they convey knowledge and skills that are essential to educated citizens, and they provide the foundations for further mathematical learning. Because the focal points are core structures that lay a conceptual foundation, they can serve to organize content, connecting and bringing coherence to multiple concepts and processes taught at and across grade levels. They are indispensable elements in developing problem solving, reasoning, and critical thinking skills, which are important to all mathematics learning.

When instruction focuses on a small number of key areas of emphasis, students gain extended experience with core concepts and skills. Such experience can facilitate deep understanding, mathematical fluency, and an ability to generalize. The decision to organize instruction around focal points assumes that the learning of mathematics is cumulative, with work in the later grades building on and deepening what students have learned in the earlier grades, without repetitious and inefficient reteaching. A curriculum built on focal points also has the potential to offer opportunities for the diagnosis of difficulties and immediate intervention, thus helping students who are struggling with important mathematics content.

What characteristics qualify a concept or topic to be a curriculum focal point? For inclusion in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, a focal point had to pass three rigorous tests:

- Is it mathematically important, both for further study in mathematics and for use in applications in and outside of school?
- Does it “fit” with what is known about learning mathematics?
- Does it connect logically with the mathematics in earlier and later grade levels?

A curriculum focal point may draw on several connected mathematical content topics described in *Principles and Standards for School Mathematics* (NCTM 2000). It should be addressed by students in the context of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation. Without facility with these critical processes, a student’s mathematical knowledge is likely to be fragile and limited in its usefulness.

A complete set of curriculum focal points, situated within the processes of mathematics, can provide an outline of an integrated mathematics curriculum that is different from the outline created by a set of grade-level mastery objectives or a list of separated content and process targets. In contrast with grade-level mastery objectives, which can be interpreted as endpoints for learning, curriculum focal points are clearly areas of emphasis, calling for instruction that will help students learn content that gives them a foundation for increasing their understanding as they encounter richer and more challenging mathematics.

Instruction based on focal points would devote the vast majority of attention to the content identified for special emphasis in a grade. A curriculum for pre-K–8 based on a connected set of such focal points could provide a solid mathematical foundation for high school mathematics.

Connecting the Standards for ***Mathematical Practice*** (***Common Core***)

to the Standards for ***Mathematical Content***

The Standards for Mathematical ***Practice*** describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical ***Content*** are a balanced combination of procedure and understanding. Expectations that begin with the word "*understand*" are often especially good opportunities to connect the *practices to the content*. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "**points of intersection**" between the Standards for Mathematical Content **and** the Standards for Mathematical Practice. These **points of intersection** are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to ***qualitatively improve*** the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

It is important to read and reflect on the above words about

Mathematical Practice Standards and Mathematical Content Standards…

Common Core State Standards for Math: Instructional programs **Pre-K to 12** should enable students to:
CCSS for Mathematics were presented in draft form as of 2010 and suggestions & comments accepted.

1 Make sense of problems and persevere in solving them: Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. **Older students might,** depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. **Mathematically proficient students** can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. **Younger students** might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, **"Does this make sense?"** They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively: Mathematically proficient students **make sense of quantities and their relationships** in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: **the ability to *decontextualize***—to abstract a given situation and **represent it symbolically and manipulate** the representing symbols as if they have a life of their own, without necessarily attending to their referents—**and the ability to *contextualize*,** to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others: Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. **They make conjectures and build a logical progression of statements to explore the truth of their conjectures**. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They ***justify conclusions, communicate them to others, and respond to the arguments of others.*** They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. **Elementary students** can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. **Later, students** learn to determine domains to which an argument applies. **Students at all grades** can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. **In early grades,** this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. **By high school**, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. **Mathematically proficient students** who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships **using such tools as** diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in context of the situation and reflect on whether results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically: Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. **Proficient students are sufficiently familiar with tools** appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. **They detect possible errors by strategically using estimation and other mathematical knowledge**. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. **Mathematically proficient students** at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision: Mathematically proficient students try to **communicate precisely to others**. They try to use clear **definitions** in discussion with others **and in their own reasoning**. They state the meaning of the symbols they choose, **including using the equal sign consistently and appropriately.** They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They **calculate accurately and efficiently**, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure: Mathematically proficient students look closely to **discern a pattern or structure**. ***Young students***, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. **Later, students** will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the **significance of an existing line** in a geometric figure and can use the strategy of **drawing an auxiliary line for solving problems**. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning: Mathematically proficient students notice if calculations are repeated, and look **both for general methods and for shortcuts.** ***Upper elementary students*** might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. ***By paying attention to the calculation*** of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. **Noticing** the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. **They continually evaluate the reasonableness of their intermediate results.**

ODE Reference: http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=1704&ContentID=83475