Synthetic Division or Synthetic Substitution for Polynomial Functions

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Definition: An **artificial (synthetic) procedure** used in place of **Long Division** or Algebraic Substitution.

Example Polynomial Function: $F(X) = 2X^3 + 3X^2 - 11X - 6$

Synthetic Division is an artificial procedure to locate **points** in an Infinite Solution Set of a Polynomial. It also is a means to determine **special points** called zero or roots or intercept points in a Solution Set. The procedure uses **detached coefficients** from the polynomial and arranges them in a horizontal row with an X value to be synthetic divided into the coefficients. The steps to this procedure and concrete examples of using the procedure are provided it the paragraph and examples that follow below.

After the horizontal arrangement has been placed to the right of any X value the **first step** is to place a 0 under the first coefficient **then add**. The **next step** is to the this **sum** and **multiply** it by the **X value** and place this **product** under the **second** coefficient **then add**. Again this **sum** is **multiplied** by the X value and the **second** product is placed under the **third** coefficient. This procedure is **repeated** until all coefficients have been added. If the **last** number is any **nonzero** number then it is a **normal** point in the Infinite Solution Set of the Polynomial Function with a Y value equal to the final number of SD. If the last number is a **zero** then it is a special point in the SS called a root or intercept point of the SS.

[+1]	+2	+3	–11	–6	Coefficients of Original Polynomial Function
0	+2	+5	–6		
+2	+5	–6	–12		An X value of +1 yields a Y value of –12

thus a normal point in SS is (+1, –12)

[+2]	+2	+3	–11	–6	
0	+4	+14	+6		
+2	+7	+3	0		An X value of +2 yields a Y value of 0

thus a special point in SS is (+2, 0)

Since the second Synthetic Division yields a 0 then the numbers in front of it are coefficients of a Depressed Equation or would be the Quotient of a normal long Algebraic Division. To determine the final two zero numbers, repeat the Synthetic Division procedure on the Depressed Equation or if the coefficients are only (3) then the Depressed Equation then it can be factored or the QRF is used.

[–1]	+2	+7	+3
0	–2	–10	
+2	+5	–7 No good	

[–2]	+2	+7	+3
0	–4	–6	
+2	+3	–3 No good	

[+3]	+2	+7	+3
0	+6	+26	
+2	+13	+29 No Good	

[–3]	+2	+7	+3
0	–6	–3	
+2	+1	0	

Therefore (–3,0) and (–1/2,0)

Now, the process of finding all the intercepts or roots or zero numbers might have been easier if factoring had been used. +2x2 +7x +3 = (X+3)(2x+1) or X=–3 and X= –1/2 Thus the special or roots or intercepts of the Polynomial Function are: (X,Y) = (+2,0) , (–3,0) and (–1/2,0). Using Synthetic Division to find a few more normal points provides enough data to sketch the Solution Set.

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