

Summary of Basic Set Theory * Page 1

A few examples of the common *numeric* sets in Mathematics are below:

Naturals, Wholes, Integers, Rationals, Irrationals, Reals...

When dealing with sets, they usually are denoted using **capital letters**.

A set is a group of objects or elements containing a common attribute.

Elements can be numbers, letters, symbols, concepts, **objects**, etc...

When dealing with sets as described above, it is common to use { }.

Note: Be able to provide adequate definitions **and** examples for below:

<http://www.mathwords.com/> Do **all** dictionaries give definitions **&** examples?

1. There are two fundamental sizes for sets which are: **Finite** and **Infinite**

Finite refers to a set of *countable* elements which might be small or large but can at least be counted in a physical or orderly manner thus ends.

Infinite set refers to a set of *non-countable* elements which are too large to be actually countable in a physical or orderly manner thus non-ending.

2. There are two basic operations on sets are **Union** and **Intersection**:

Union means to join together the elements of two sets, however, elements are not to be duplicated or repeated in the combined or total set(s).

Example of Union: $A = \{1,2,3,4\}$ $B = \{3,4,5,6\}$ $A \cup B = \{1,2,3,4,5,6\}$

Intersection means to determine the common elements shared by two or more sets however do not duplicate or repeat in overlapping set(s).

Example of Intersection: $A = \{1,2,3,4\}$ $B = \{3,4,5,6\}$ $A \cap B = \{3,4\}$

3. Two special sets arise from Set Theory (*Operations*): **Universe** and **Null Sets**.

The **Universe Set** can be somewhat confusing so let's take a closer look at Universe:

Universe **{U}** can be finite to infinite and always contains all elements to be considered.

Examples: $\{0,1,2,3,4,5,6,7,8,9\}$, Double Digits, Evens, Odds, Naturals, Integers, Reals...

The **Null Set** is rather simple because it contains nothing, not even 0. Null Symbol = \emptyset

When dealing with sets, it is important at the beginning to declare: The Universe Set.

4. Two more special sets arise from Universe and Null Sets called **Complements**.

Example: $U = \{1,2,3,4,5,6,7,8\}$ $X = \{1,2,3,4\}$ $Y = \{5,6,7,8\}$ X & Y contain all in **U** thus X & Y are complements. **Criteria:** $X \cup Y = U$ & $X \cap Y = \text{Null} = \emptyset$

The **Prime Symbol** is used to denote complements: X & $X' = Y$ Y & $Y' = X$

5. Two more special types of sets arise from Set Theory called **Subset** and **Superset**.

Subset is a smaller set which is part of another set **and** not equal **but** contained within.

Example: Subsets of P = $\{0,1,2,3,4,5,6,7,8,9\}$ **Q** = $\{2, 4, 6, 8\}$ **R** = $\{1, 3, 5\}$ **S** = $\{1\}$

Another example of Subset with larger sets: The Naturals are a subset of Integers.

Superset is a larger set which is but more specific is a set that contains another set.

Example of Supersets: The Reals are a Superset of Integers $\{\dots,-2, -1, 0, +1, +2,\dots\}$

Another example is: The Integers are a Superset of Wholes $\{0, 1, 2, 3,\dots\}$ **Ws \neq Ns**

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Subsets are smaller sets which are part of others **and** not equal **but** contained within.

Example: Subsets of A = {Letters of Alphabet} **V** = {Vowels} **C** = {Consonants}

Another example of Subset with larger sets: The Evens & Odds are subsets of Naturals.

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6. Sets must be considered in two basic and common ways: **Equal & Non-equal**

Equal sets are collection of objects that must have an equal amount of objects (elements) **however** they do not have to have same order.

Examples: $\{a, b, c\} = \{c, a, b\}$ $\{ @, \#, \&, \% \} = \{ \&, \% , @, \# \}$

Non-equal sets are collection of objects that do not have equal amounts of objects (elements) and again they do not have to have same order.

Examples: $\{a, b, c\} \neq \{a, b\}$ $\{ @, \#, \&, \% \} \neq \{ \&, \% , @, \#, \$ \}$

7. Subsets may be thought of as similar **yet** different types: **Proper & Improper**

Proper subsets are usually thought of as *smaller sets* than an initial set or **U** set *however* Proper subsets must be *contained within* the initial set or **U** set.

Examples: Alphabet = $\{a, b, c, d, \dots\}$ SDs = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Evens = $\{0, 2, 4, 6, 8, 10, 12, \dots\}$ Odds = $\{1, 3, 5, 7, 9, 11, 13, \dots\}$

Challenge: Can you describe these examples using specific descriptive words?

The **Null Set** is a *special case* of Proper Subsets **but** must be considered nonetheless.

Improper subsets are often times thought of as *equal sets* to an initial set or **U** set but also *critical* is Improper subsets must be *contained within* the initial set or **U** set.

Examples: {Naturals} are an improper subset of {Wholes} equal to **&** contained +Odds are *Improper* to Integers Naturals are *Improper* to Reals

Challenge: Consider the concept of **Infinite** sets and discuss/explain above examples.

The **Universe** is a *special case* of Improper Subsets **and** must be considered as such.

8. The number of subsets (Proper & Improper) can be found with a simple equation.

Total number of Subsets to any finite set is equal to: 2^n **n = number of elements.**

Example: $U = \{1, 2, 3, 4\}$ Subsets = $2^4 = 16$ **Null** = $\{\}$, **1's** = $\{1\}, \{2\}, \{3\}, \{4\}$

2's = $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$ **3's** = $\{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 4\}, \{2, 3, 4\}$ **4's** = $\{1, 2, 3, 4\}$

Thus subsets can be arranged in an orderly manner: 1, 4, 6, 4, 1 = 16 **Interesting...**

Challenge: Find all Subsets for $\{a, b, c\}$ & $\{ @, \#, \% , \&, \$ \}$ and show in orderly manner.

Do you see any areas of Math **overlapping**? Pascal's Triangle, Combinations, others...

<http://betterexplained.com/> Neat site trying to help others **with simple** explanations.